Real Options

A. Puts and Calls

Options are the right but not the obligation to enter into a pre-specified transaction at some time in the future.

Call option – the right to buy an asset at a pre-specified price.

Put option – the right to sell an asset at a pre-specified price.

Projects often have option-like components. For example, there is the option to postpone investment, or to invest more if the project goes well.

We will therefore look at the components of option value.

A.1. Call Option

A call option is the right to buy an asset at a given price. Suppose a call offers the right to buy asset S at time T for the strike price K. Then the payoff of a call option is:

$$C(T) = \max(S_T - K, 0)$$

If the price of the asset at time T is greater than the strike price, the holder of the option will exercise his or her right, and buy the asset for the strike price. If the price at time T is lower than the strike price, the holder of the option will allow the option to expire.

A *European* call option may only be exercised at time T; an *American* call option may be exercised at any time up until T.

What should the value of a call option be prior to time T? Under several assumptions, the most important of which is that the volatility of the underlying asset is constant, Black and Scholes and Merton derived an option pricing formula:

$$C(t,T)=\left[\begin{array}{l} S\bullet N\left(\dfrac{\ln\dfrac{S}{K}+\left(r+\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \\ -K\bullet e^{-r(T-t)}\bullet N\left(\dfrac{\ln\dfrac{S}{K}+\left(r-\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\end{array}\right]$$

where r is the instantaneous interest rate (assumed to be constant), and s is the instantaneous volatility of the underlying asset. The N function is the cumulative distribution function of the standard normal:

$$N(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x}e^{-\frac{u^2}{2}}du$$

A European call option cannot be exercised prior to time T. Although an American call option can be, it is never optimal to exercise early, provided the underlying asset pays no dividends.

Why is it not optimal to exercise an option early? There are two components to the option value:

(1) Intrinsic value – the difference between the price of the underlying asset and the strike price.

(2) Option value – value that comes from the ability to delay the decision on whether to purchase the asset.

Both of these components are more valuable if we wait to exercise the option. The option value disappears as soon as we exercise; if the price of the asset subsequently goes down, we can no longer simply allow the option to expire. Note that intrinsic value is also higher if we wait; exercising now rather than later means we have to pay the strike price now rather than later, so its present value is higher.

Since it is never to exercise an American call option prior to expiry (when the asset does not pay dividends), the price of an American call option is equal to the price of the European call option.

When the asset pays dividends, the situation is a little different. For example, suppose the asset pays a continuous dividend yield at rate δ. Then the European option price is:

$$C(t,T)=\begin{bmatrix} S\bullet e^{-\delta(T-t)}\bullet N\left(\dfrac{\ln\dfrac{S}{K}+\left(r-\delta+\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \\ -K\bullet e^{-r(T-t)}\bullet N\left(\dfrac{\ln\dfrac{S}{K}+\left(r-\delta-\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right) \end{bmatrix}$$

When the asset pays a dividend, it may be worthwhile to pay the strike price today and give up the option value in order to begin to receive the dividend. Pricing of American options is far more difficult when the asset pays a dividend.

A.2. Put Option

A put option is the right to sell an asset at a pre-specified price. The payoff of a put option is:

$$P(T)=\max\left(K-S_T,0\right)$$

The price of a put option is (under the same assumptions needed in the case of a call option, most importantly, constant volatility):

$$P(t,T)=\left[\begin{array}{l} K\bullet e^{-r(T-t)}\bullet\left[1-N\left(\dfrac{\ln\dfrac{S}{K}+\left(r-\delta-\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\right] \\ -S\bullet e^{-\delta(T-t)}\bullet\left[1-N\left(\dfrac{\ln\dfrac{S}{K}+\left(r-\delta+\dfrac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}\right)\right]\end{array}\right]$$

The optimal exercise strategy of an American put option is subtler than that for a call option. For a put option, the holder receives rather than pays the strike price, so one of the two factors (assuming no dividends) determining option value causes us to want to exercise the option early (i.e., the present value of the strike price is higher if we receive it right away). Therefore, even when there is no dividend, there may be cases where it is optimal to exercise a put option early.

B. Arbitrage Pricing of Options

How are the option pricing formulae shown above derived? The Black-Scholes-Merton option pricing formula, in 1973, was the first application of a dynamic arbitrage strategy. It can be shown (we will not) that the payoff of an option can be replicated by forming a portfolio containing the underlying asset and a risk-free bond. However, there is no static arbitrage, i.e., the contents of this replicating portfolio must continually be updated. The replicating strategy is self-financing, i.e., no additional funds must be added to or taken from the replicating portfolio as it is updated. The price of the option is the initial value of the replicating portfolio.

Why can we not simply apply the approach we have taken in capital budgeting, i.e., discount the expected cash flows? The problem with this approach is that the appropriate discount rate for an option is not constant. When the option is deeply in the money (i.e., virtually certain to be exercised), it is roughly as risky as the underlying asset. On the other hand, when the option is deeply out of the money (i.e., virtually certain not to be exercised), the option has almost no risk (and almost no value). So as the asset price moves up and down, the risk of the option changes, and the appropriate discount rate also changes. So we cannot simply discount the expected cash flows at some discount rate.

C. Volatility

Option value is driven both by the price of the underlying asset and its volatility. Both put and call options are more valuable when the underlying asset is more volatile. Why?

This property of options has important implications for corporate finance. As we shall see, the volatility of a firm's assets, even holding their present value constant, has an effect on the value of both debt and equity. Furthermore, the volatility of a project may change its value in important ways.

D. Corporate Securities as Options

When Black and Scholes published their famous (and now Nobel-prize winning) article in 1973, they thought they were valuing stocks and bonds, not options. Why is that?

Consider the value of a firm's assets. Taking a somewhat simplified view of debt, let us suppose that debt must all be paid off at a time T. At time T, if the value of the firm's assets are greater than the face value of the debt, the debtholders will be receive the full amount they are owed. However, if the value of the firm's assets are less than the face value of the debt, the equityholders have the option to have the firm declare bankruptcy. In this case, the firm's assets are turned over to the debtholders, who therefore receive only a partial repayment of their debt.

The equity is therefore similar to a call option – the equityholders have the option to buy the assets of the firm, by paying off the debtholders for a fixed amount.

The debt is similar to a combined position of risk-free debt, and a short put option. That is, although the debtholders are owed a certain amount of money, the equityholders have the right to sell them the firm's assets in exchange for the same amount of money.

Consider a zero-NPV project that has a lot of volatility. Doing this project does not increase the value of the firm's assets. Does it affect the value of the debt and equity?

Does this have any effect on the incentives of the management of the firm?

Firms often have several grades of debt. For example, *senior* debtholders have a claim to the firm’s assets, that must be satisfied before *junior* debtholders are paid off. Can you express the value of junior debt in terms of options?

E. Embedded Options

Projects can often be thought of in option terms. Suppose a project requires an initial investment, and then generates some positive cash flows. We can think of the project as the right (but not the obligation) to purchase the cash flows of the project for the initial investment. What is the optimal time to exercise this option?

E.1. Option to Wait

Consider the option to invest in a gourmet coffee shop in Princeton, New Jersey. The cash flows of this project will depend on both the state of the economy, and the preferences of Princeton coffee consumers.

Suppose the risk-free rate is 2.1%, and the market risk premium is 8.4%. The beta of an investment in gourmet coffee shops is 0.8333%.

Market Type	Probability	Demand	Probability	Cash Flow
Berkeley	0.8	High	0.25	$32
		Medium	0.5	$22
		Low	0.25	$12
Detroit	0.2	High	0.25	$22
		Medium	0.5	$12
		Low	0.25	$2

The cash flows are annual for ten years, and the initial investment required is $100. What is the NPV of this project?

$$E[R] = R_f + \beta\left(E[R_M] - R_f\right)$$

$$= 2.1\% + 0.8333 \bullet 8.4\% = 9.1\%$$

Note that, if Princeton resembles the Berkeley market, the expected cash flow is equal to \$22. If Princeton resembles the Detroit coffee market, the expected cash flows are \$12. Since there is an 80% chance of a Berkeley-like market and a 20% chance of a Detroit-like market, the expected annual cash flow without knowing which market Princeton most resembles is \$20 per year. The NPV of the project is then:

$$NPV = -\$100 + \sum_{i=1}^{10} \frac{\$20}{(1+9.1\%)^i} = \$27.8$$

Should you do the project? Is it possible to do better?

Certainly doing a positive NPV project such as this one is better than not doing it, but maybe we can structure the problem so that we can achieve a higher NPV still. Suppose we can hire a market research firm that will tell us what the Princeton coffee market is like (i.e., whether it is like Berkeley or Detroit). The study takes one year. How much should we be willing to pay for this market research?

Suppose that the study tells us that the Princeton coffee market is like Detroit. Conditional on this information, what is the NPV of the project (one year from now)?

$$NPV = -\$100 + \sum_{i=1}^{10} \frac{\$12}{(1+9.1\%)^i} = -\$23.2$$

Clearly, if the study tells us that the Princeton coffee market is similar to the Detroit coffee market, we will not open the store. (Note that we are implicitly assuming that the beta of the cash flows is related to general economic conditions, rather than the nature of the Princeton market.) What if the Princeton coffee market is like Berkeley?

$$NPV = -\$100 + \sum_{i=1}^{10} \frac{\$22}{(1+9.1\%)^i} = \$40.6$$

This study cost us a year delay. So in the 0.8 probability case that the Princeton coffee market is like the Berkeley Coffee market, the NPV of the project (excluding the cost of the market research) is:

$$NPV = \frac{(0.8)\bullet(\$40.6)+(0.2)\bullet(\$0)}{1+9.1\%} = \$29.77$$

How much should we be willing to pay for this market research?

E.2. Option to Expand

Suppose that in 1992 you can invest in an internet bookstore. The investment costs \$100M, and produces cash flows of \$11M each year if people like buying books on-line, and \$0 if they don't. Each scenario has probability 0.5. The discount rate is 10%. Should you do the project?

If people like buying books on-line, then the cash flows received are a perpetuity:

$$PV = \frac{CF}{R} = \frac{\$11M}{10\%} = \$110M$$

However, there is only a 50% chance of receive this $110M cash flow, so the present value of the expected cash flows is only $55M. The net present value of the project is negative $45M; clearly a loser.

Now suppose you can wait one year to observe the market and determine whether people like to buy books on-line. What should you do?

One year from now, you should do the project if people like buying books on-line. In this case, the project will be a $10M positive NPV project. If people don't like buying books on-line, then don't make the investment. Since there is a 50% chance of receiving net cash flows worth $10M in present value one year from now, and a 50% chance of receiving nothing one year from now. So the net present value of this strategy is:

$$NPV = \frac{0.5 \bullet (\$10M) + 0.5 \bullet (\$0)}{1 + 10\%} = \$4.55M$$

(Is 10% the appropriate discount rate to use in this case? Could you make an argument for some other rate? Do you incur any risk during the first year?)

So it would seem that the optimal strategy here is to wait one year and make the investment then, provided that people like to buy books on-line.

Might there ever be a case in which you would want to go ahead and make the investment today anyway? Suppose that being the first on-line bookstore provides you with a brand image that gives you an advantage in other markets. For example, if you decide to sell VHS tapes, CDs, or other products on-line, being the first on-line bookstore might help you enter these markets more easily. If so, then you have an option if you make the investment today. When you invest the $100M today, you acquire not only a 50% chance of receiving annual cash flows of $11M, but also a competitive advantage in other markets, which has value. The value of this option ought to be included in the original analysis.

E.3. Option to Open or Close

Consider a gold mine. You can purchase the right to manage the gold mine for the next three years. Each year, the mine can produce 50,000 ounces of gold. Costs of extraction are $230 per ounce, and the price of gold is $220 per ounce. With equal probability, the price of gold will rise 20% or fall 10% in each of the next two years. If the appropriate discount rate is 5%, what should you do?

We might be inclined to perform the following (incorrect) analysis. The expected value of gold in the second year is:

$$E\left[P_2\right]=\begin{bmatrix}0.5\bullet(1+20\%)\bullet\$220\\+0.5\bullet(1-10\%)\bullet\$220\end{bmatrix}=\$231$$

In the third year, the expected value of gold is:

$$E\left[P_3\right]=\begin{bmatrix}0.5\bullet(1+20\%)\bullet\$231\\+0.5\bullet(1-10\%)\bullet\$231\end{bmatrix}=\$242.55$$

Since gold costs \$230 per ounce to mine, and the mine can produce 50,000 ounces per year, the expected cash flows are:

$$E[CF_1] = 50{,}000 \bullet (\$220 - \$230) = -\$500{,}000$$
$$E[CF_2] = 50{,}000 \bullet (\$231 - \$230) = +\$50{,}000$$
$$E[CF_3] = 50{,}000 \bullet (\$242.55 - \$230) = +\$627{,}500$$

Discounting these cash flows at the 5% rate, we find:

$$PV = \frac{-\$500{,}000}{1+5\%} + \frac{\$50{,}000}{(1+5\%)^2} + \frac{\$627{,}500}{(1+5\%)^3} = \$111{,}219$$

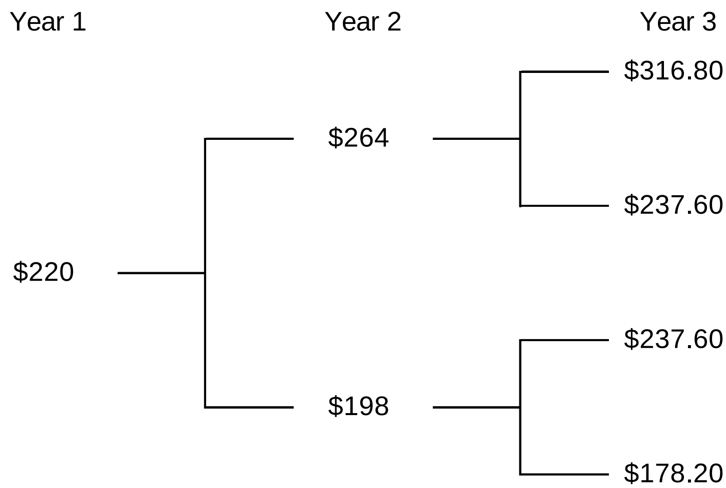
So if we are able to purchase the rights to the mine for three years for less than \$111,219, the project will have a positive NPV.

What is wrong with the above analysis? Note that in the first year, we are mining 50,000 ounces of gold, even though it is unprofitable to do so. Suppose we close the mine, and open it only in the second and third years, when the expected price of gold is higher than the extraction cost? In that case, the (still incorrect) present value is:

$$PV = \frac{\$50{,}000}{(1+5\%)^2} + \frac{\$627{,}500}{(1+5\%)^3} = \$587{,}410$$

By cutting out the first years production, we have improved the present value of the cash flows from the project by a factor of more than five.

However, we still haven't analyzed this project correctly. Rather than committing today to open the mine in the second and third years, why not wait until we know the price of gold, and open the mine only if it is higher than the extraction cost? The following tree shows the possible paths that the price of gold could take:



Each of the possible year 2 prices occurs with 50% probability, and each of the year 3 possible prices occurs with 25% probability. In the second year, we will only open the mine if the price is $264; it is unprofitable to mine at $198. In the third year, we will open the mine only if the price of gold is $316.80 or $237.60. The expected cash flows are then:

$$E[CF_1] = \$0$$

$$E[CF_2] = 0.5 \bullet 50,000 \bullet (\$264 - \$230) = \$850,000$$

$$E[CF_3] = \begin{bmatrix} 0.25 \bullet 50,000 \bullet (\$316.80 - \$230) \\ +0.25 \bullet 50,000 \bullet (\$237.60 - \$230) \\ +0.25 \bullet 50,000 \bullet (\$237.60 - \$230) \end{bmatrix}$$

$$= \$1,275,000$$

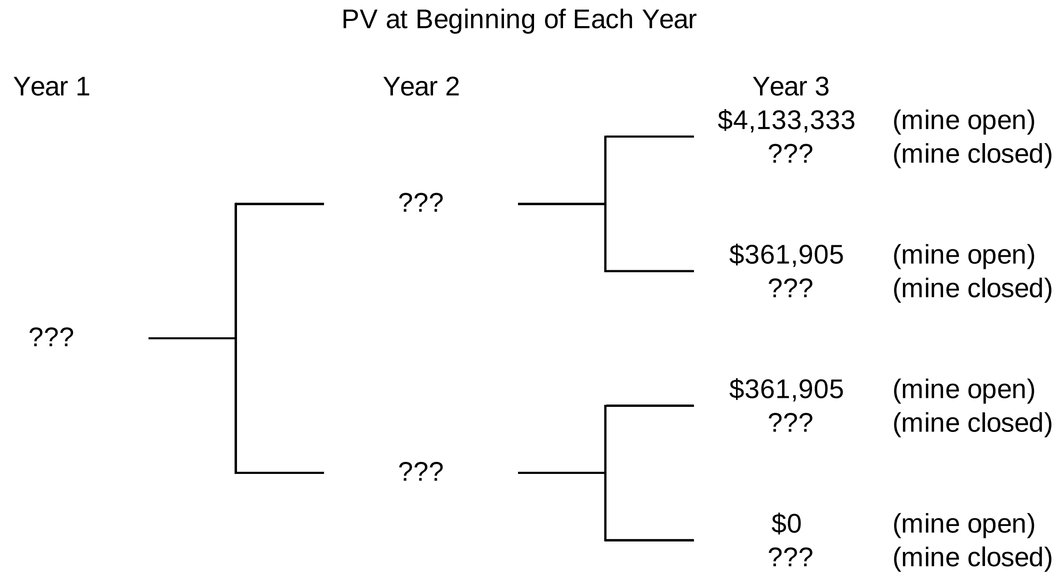
Discounting these at the rate of 5%, we find the present value of the cash flows:

$$PV = \frac{\$0}{1+5\%} + \frac{\$850,000}{(1+5\%)^2} + \frac{\$1,275,000}{(1+5\%)^3} = \$1,965,987$$

So, provided you are able to open and close the mine at will (and at no cost), the present value of the cash flows are more than 17 times higher than if you simply kept the mine open regardless of the price of gold.

In the above analysis, we assumed that it is costless to open or close the mine at will. Suppose it really is costless to close the mine, but that an investment of $500,000 is required to reopen the mine once it is closed. How does this change the analysis?

We will determine the present value of this project by working backwards. Suppose that at the beginning of the third year, the mine is open. Given the prices in the tree above, we can calculate the cash flows in each of the four states. If we discount them one year, we have their present values at the beginning of year 3:



Suppose, however, that the mine is closed at the beginning of year 3. Will you pay $500,000 to reopen it? When the price of gold is $316.80, the present value (at the beginning of year 3) of reopening the mine is:

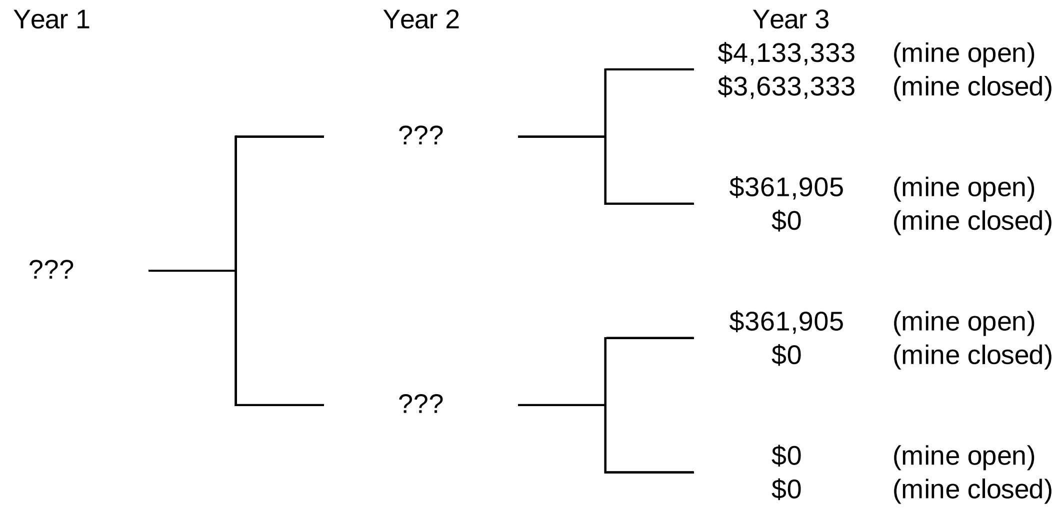
$$PV = -\$500,000 + \frac{\$4,340,000}{1.05} = \$3,633,333$$

So clearly you will be willing to reopen the mine if the price of gold is $316.80. What if it is $237.60? Then the present value is:

$$PV = -\$500,000 + \frac{\$380,000}{1.05} = -\$138,095$$

So if the mine is closed and the price of gold is only $237.60, you should not reopen the mine. The following table then shows the present value of the mine in both cases:

PV at Beginning of Each Year



Now consider year 2. If the mine is already open, and the price of gold is $264, the present value of leaving the mine open (at the beginning of the year 2) is $1,700,000 ($34 per ounce times 50,000 ounces) received in Year 2, plus a 50% chance of receiving $4,133,333 PV at the beginning of the third year, and a 50% chance of receiving $361,905 instead. These cash flows must be discounted one more year if we want to find the present value at the beginning of year two:

$$\begin{aligned} PV &= \frac{\$1,700,000 + 0.5 \bullet \$4,133,333 + 0.5 \bullet \$361,905}{1.05} \\ &= \$3,759,637 \end{aligned}$$

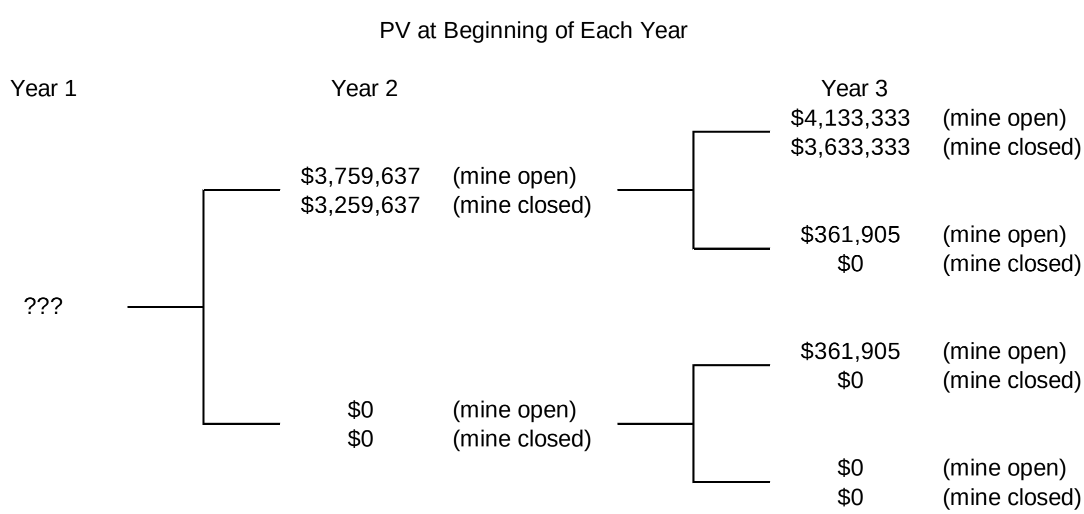
It should be obvious that this is preferable to closing the mine if the price of gold is $264.

What if the price of gold is $198? If we leave the mine open, we receive a cash flow of negative $1,600,000, and a 50% chance of a positive cash flow of $361,905. Discounting these one more year, we find the present value at the beginning of year 2:

$$
\begin{aligned}
PV &= \frac{-\$1,600,000 + 0.5 \bullet \$361,905 + 0.5 \bullet \$0}{1.05} \\
&= -\$1,351,474
\end{aligned}
$$

On the other hand, we can close the mine at the beginning of year 2 if the price of gold is $198, in which case we receive no cash flow in year 2, and a present value of year three cash flows of $0. So if the price of gold is $198, we should close the mine if it is open.

We now have:

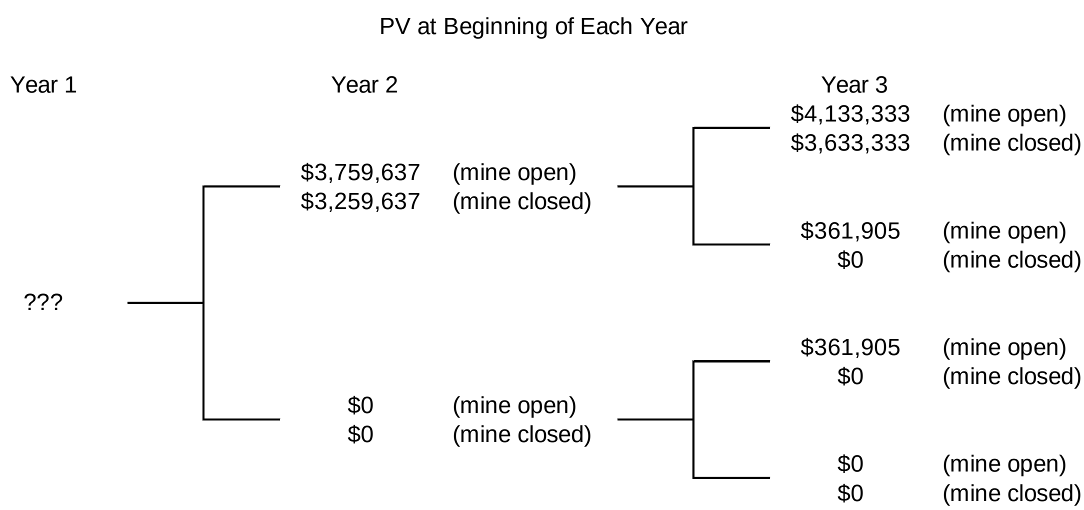


What if the mine is closed at the beginning of Year 2? It should be clear that, if the price of gold is $198, we will not open the mine, and the present value (at the beginning of

Year 2) is $0. But what if the price of gold is $264? Will we reopen the mine? The present value in this case is the $3,759,637 calculated above, minus the $500,000 cost of reopening the mine, or $3,259,637. On the other hand, if we leave the mine closed, we receive no cash flow in year 2, and a 50% chance of receiving $3,633,333. The present value of this option is:

$$PV = \frac{0.5 \bullet \$3,633,333}{1.05} = \$1,730,159$$

It is clearly preferable to open the mine. We now have:



What should we do at the beginning of year 1? If we leave the mine open, we receive a negative cash flow of $500,000, and a 50% chance of receiving $3,759,637. The present value at the beginning of year 1 is then:

$$PV = \frac{-\$500,000 + 0.5 \bullet (\$3,759,637)}{1.05} = \$1,314,113$$

If we close the mine, we receive a 50% chance of receiving $3,259,637:

$$PV = \frac{0.5 \bullet (\$3,259,637)}{1.05} = \$1,552,208$$

So, it is best to close the mine at the beginning of year 1. The optimal strategy, with present values, is shown below: